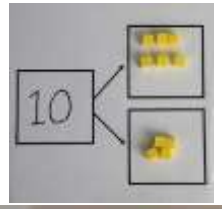
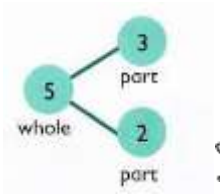
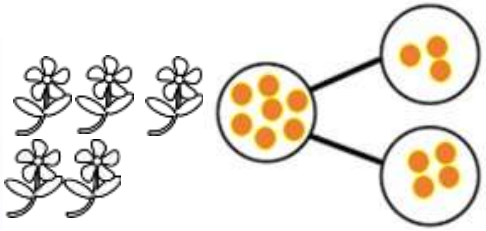
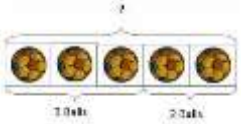
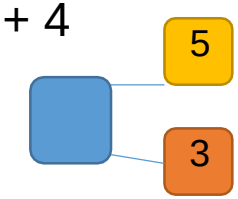
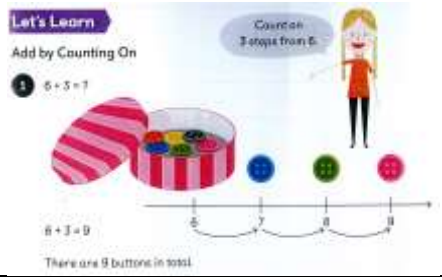
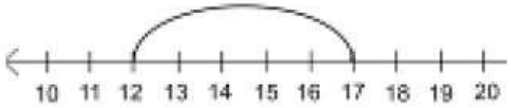
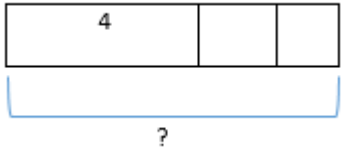
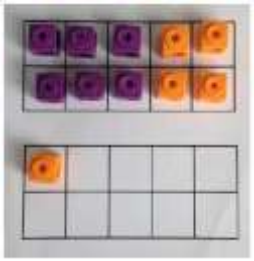
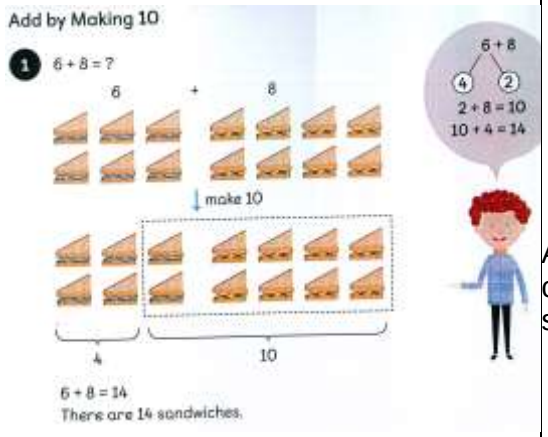
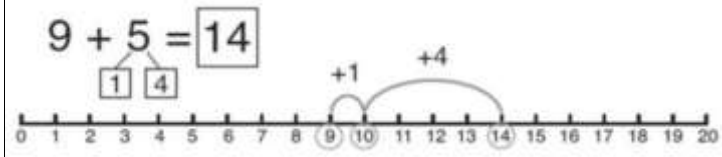
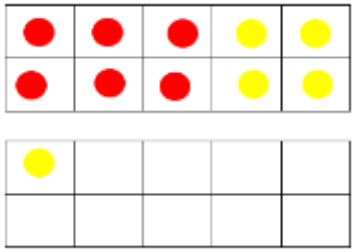
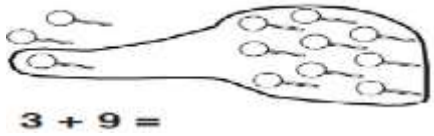
FCVS
Mathematics Calculation Policy

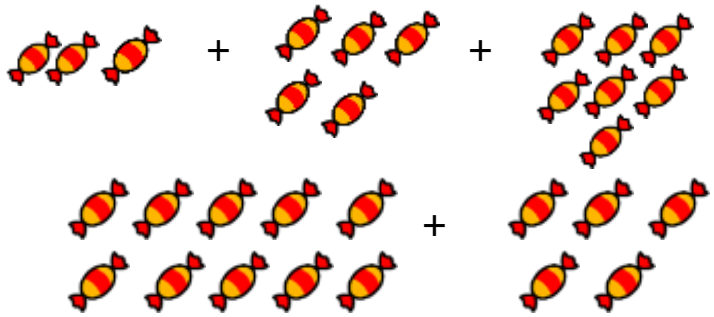
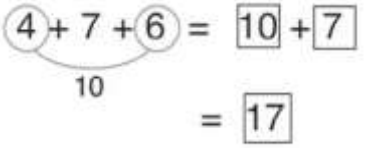
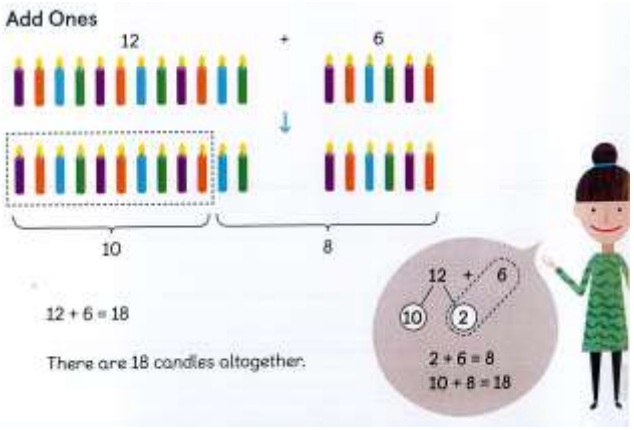
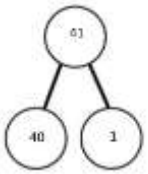
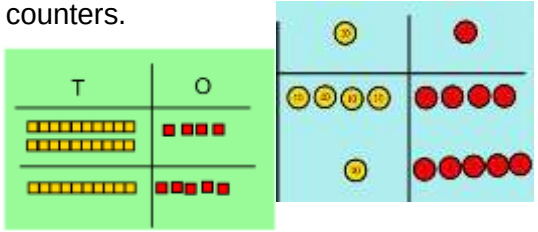
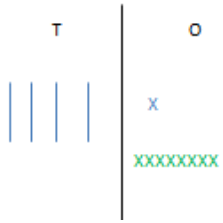
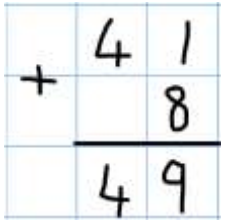
May 2021

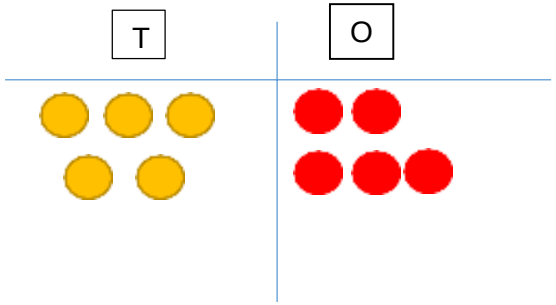
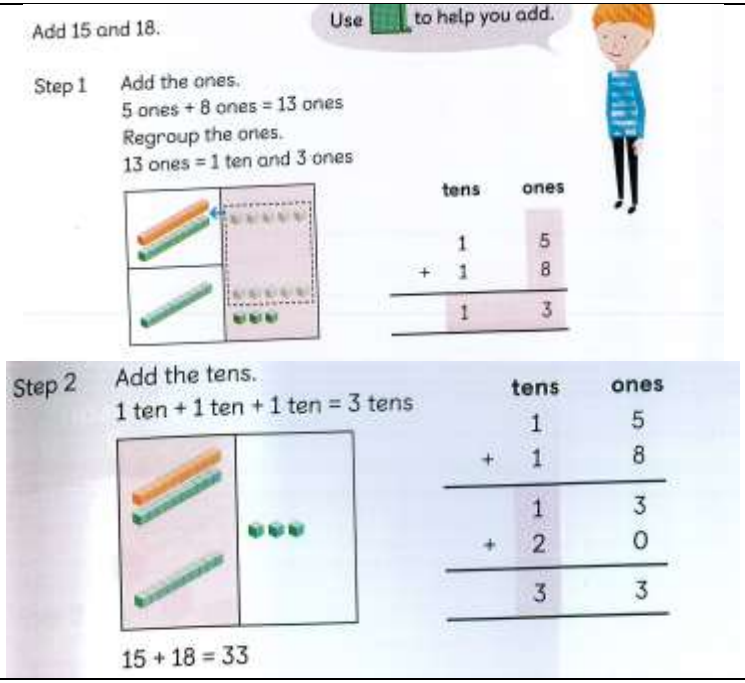
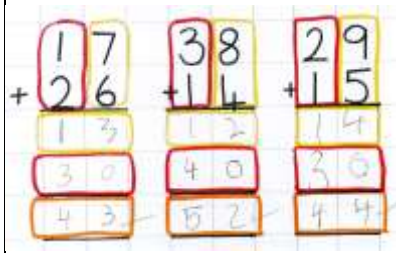
Progression in Calculations

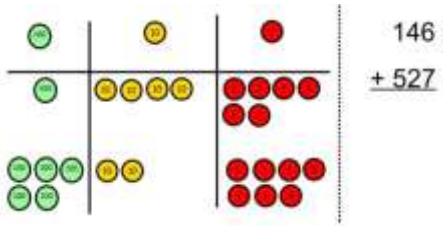
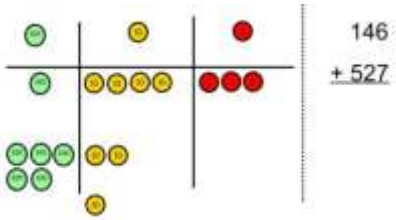
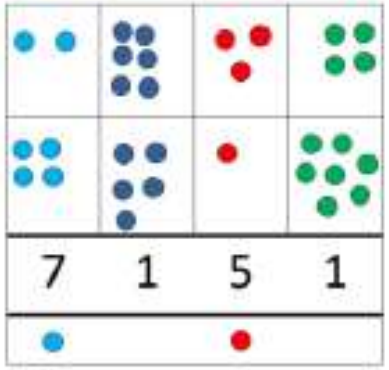
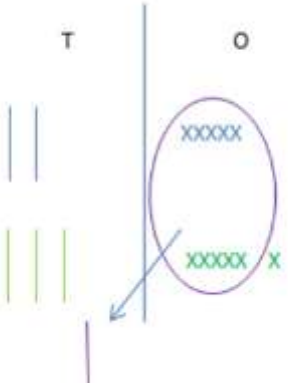
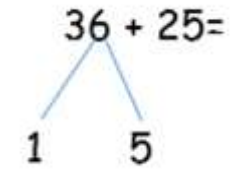
Addition

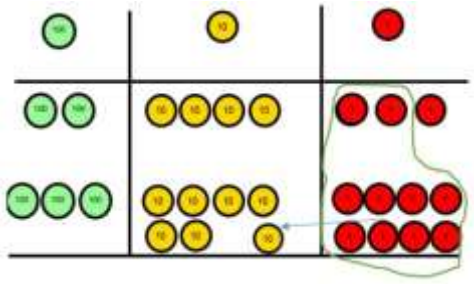
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar.  (use other resources too e.g. eggs, shells, teddy bears etc)	   Use pictures to add two numbers together as a group or in a bar. 	(4 is a part, 6 is a part and the whole is 10) 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on the smaller number 1 by 1 to find the answer. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.  A bar model which encourages the children to count on	4 + 2 = 6 Place the larger number in your head and count on the smaller number to find your answer.  The abstract number line: What is 2 more than 4? What is the sum of 4 and 2? What's the total of 4 and 2? 4 + 2

Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10 by using ten frames and counters/cubes or using numicon. 	Use pictures or a number line. Regroup or partition the smaller number to make 10.   Children to draw the ten frame and counters/cubes  After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Children to develop an understanding of equality e.g 6 + ☐ = 11 or 6 + 5 = 5 + ☐ or 6 + 5 = ☐ + 4

Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Adding ones to a two digit number	Using objects split the two digit number into the tens and ones and add the ones to the ones before recombining.		13 + 4 = 17 10 3+4 = 17 41 + 8 1 + 8 = 9 40 + 9 = 49 
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	 Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.	

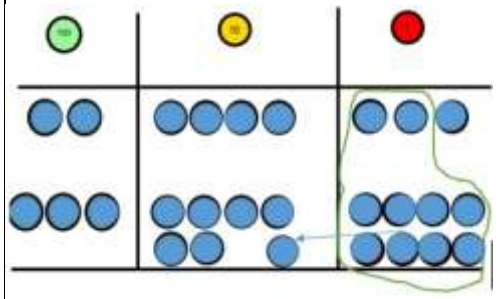
	Continue to develop understanding of partitioning and place value 41 + 8 	 After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.	Calculations 21 + 42 = 21 + 42
Expanded column addition method	Use concrete objects such as sweets to add together grouping them together in tens and ones first to make adding easier. Regroup the ones into a group of ten and ones then adding the ones followed by the groups of ten.	 	

Column method- regrouping	Make both numbers on a place value grid.  Add up the units and regroup 10 ones into one 10.  Add up the rest of the columns, regrouping the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.   	Looking for ways to Make 10:  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Start by partitioning the numbers before moving on to clearly show the regrouping below the addition. 20 + 5 40 + 8 60 + 13 = 73 And then the expanded method: 25 + 48 13 60 73 Formal method: 36 +25 61 1 536 + 85 621 1 1



Once the children have had practice with this, they should be able to apply it to larger numbers and the abstract.

Chidren to represent the counters e.g. like the image below:



If the children are completing a word problem, draw a bar model to represent what it's asking them to do.

?	
243	368

Expanded method:

243
+ 368
11
+ 100
500
611

Standard written method:

243
+368
611
1 1

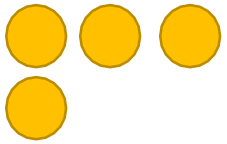
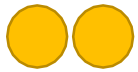
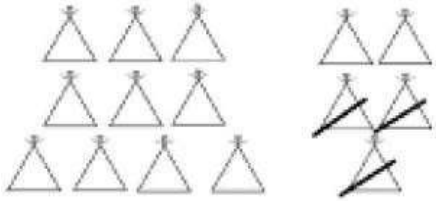
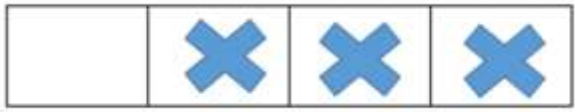
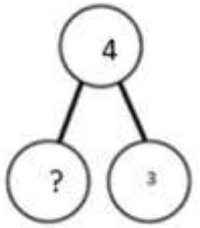
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

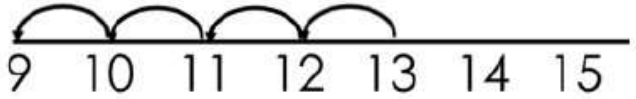
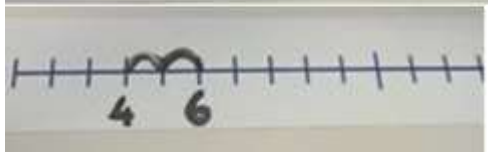
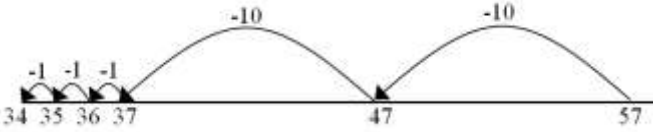
72.8
+ 54.6
127.4
1 1

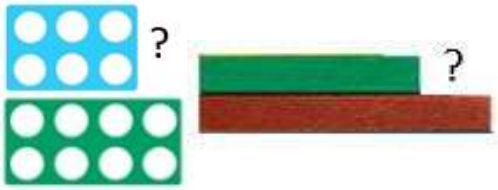
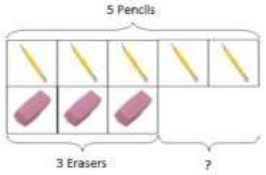
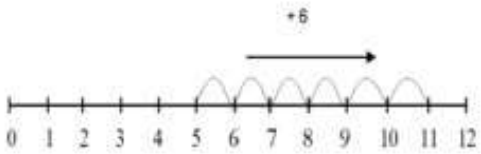
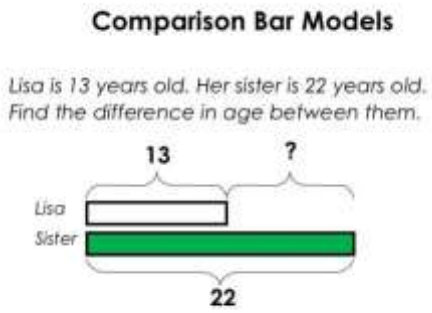
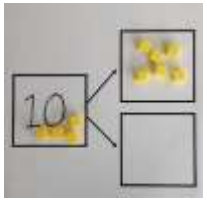
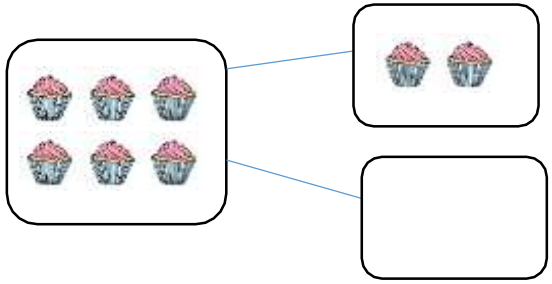
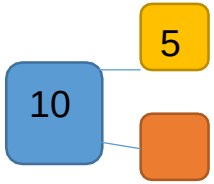
£ 2 3 . 5 9
+ £ 7 . 5 5
£ 3 1 . 1 4
1 1 1

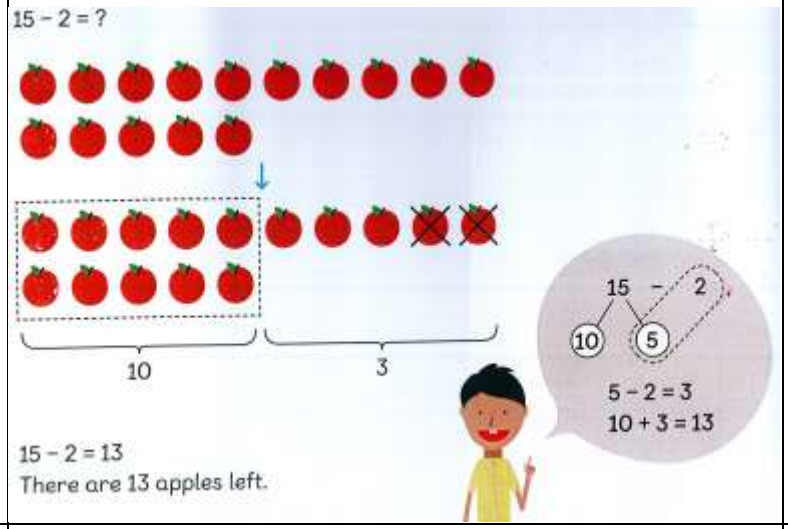
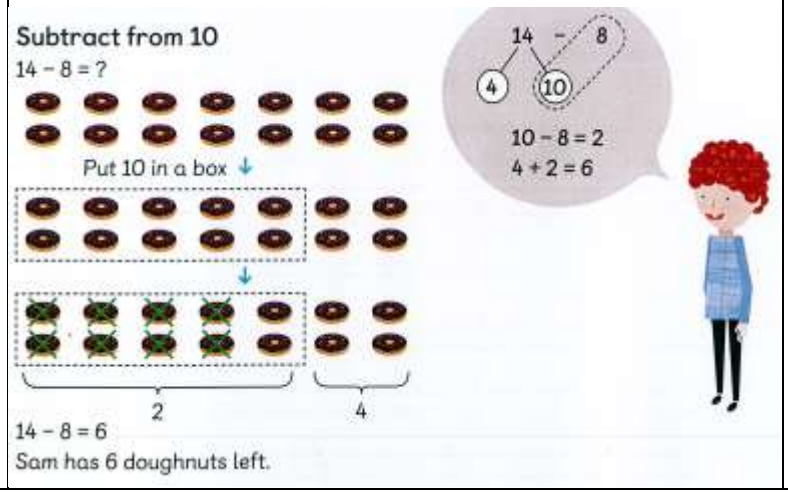
2 3 . 3 6 1
9 . 0 8 0
5 9 . 7 7 0
+ 1 . 3 0 0
9 3 . 5 1 1
2 1 2

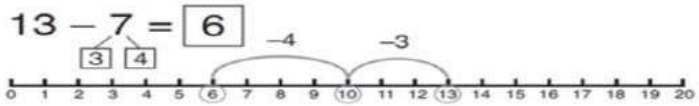
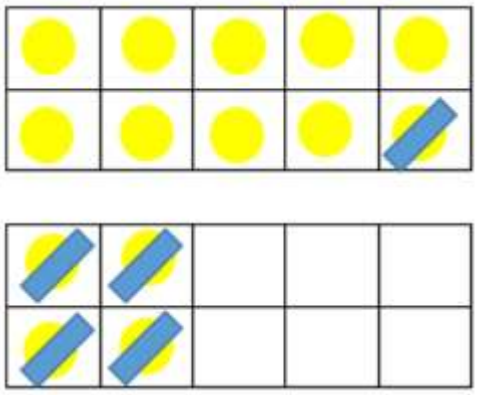
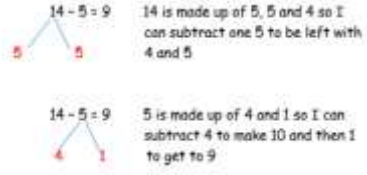
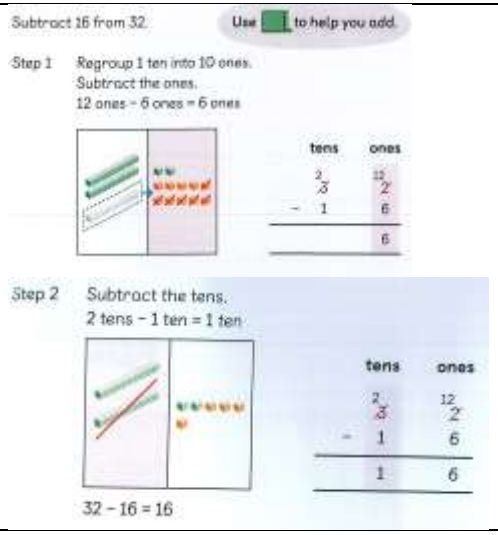
Subtraction

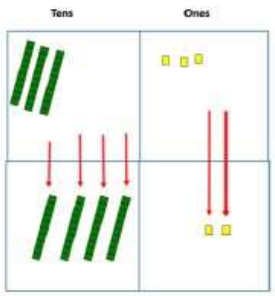
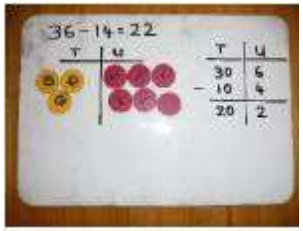
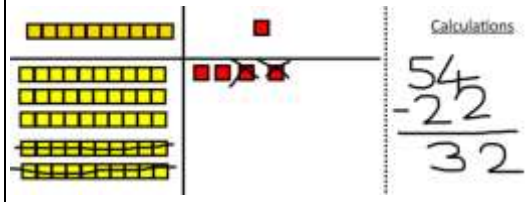
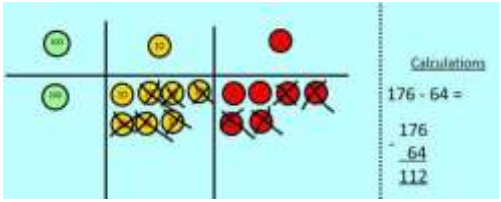
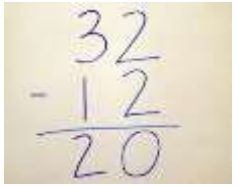
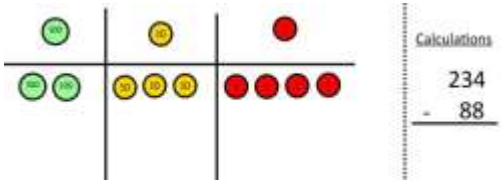
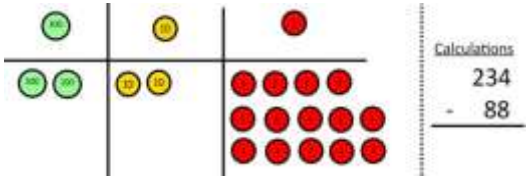
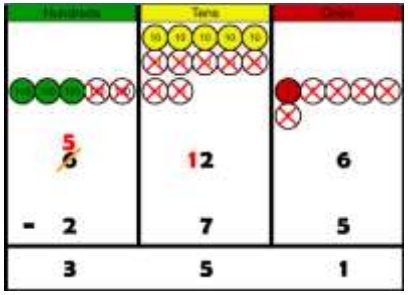
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12  Use of the bar model to support understanding: 	18 - 3= 15 8 – 2 = 6 **4- 3 =** **= 4 - 3**  

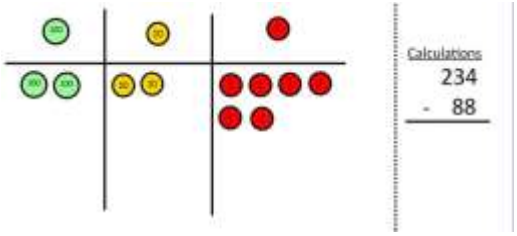
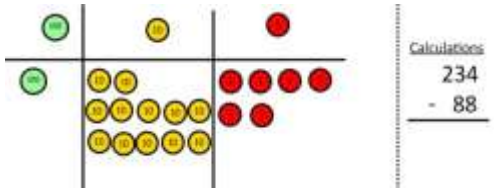
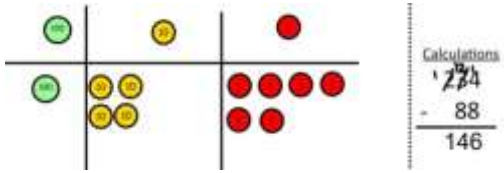
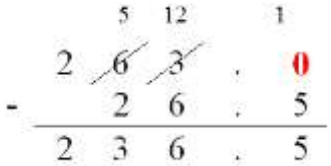
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go.  Using number lines or number tracks 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.    This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Find the difference	Compare amounts and objects to find the difference (use cubes, numicon or Cuisenaire rods.)  Use cubes to build towers or make bars to find the difference   Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  	Find the difference between 8 and 6. 8 – 6, the difference is? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987) Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the numbers of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

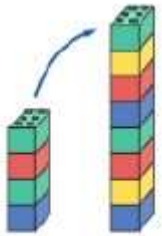
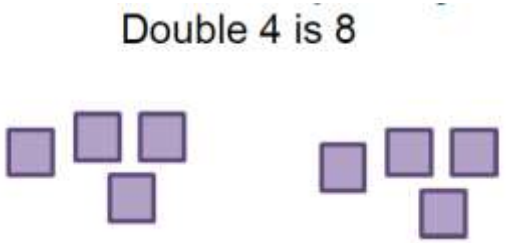
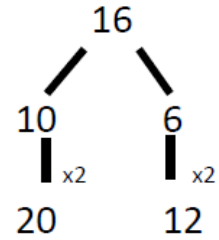
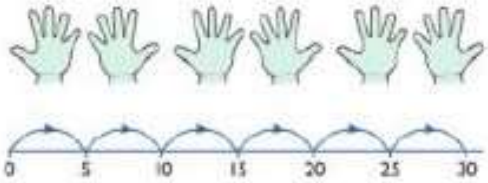
Subtract by subtracting ones	Start by using objects on ten frames so that the 10 can be seen as separate and then take away from the ones.		17 – 5 = ? 10 + (7 - 5) = 10 + 2 = 12
Subtract from 10	Start by using objects on ten frames so that the 10 can be seen as separate and then take away from the ten first.		15 – 7 = ? (10 – 7) + 5 = 3 + 5 = 8

Make 10	14 – 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Children can represent the ten frame pictorially. 	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children should represent how they have solved the calculation.  16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Expanded method for subtraction with regrouping	Work with concrete objects first – grouping them into tens and ones. Discuss whether you can take away six ones if there are only two. Then model taking a group of ten to join the two. Now you can take away six. Then subtract one group of ten from the remaining two groups of ten.	 	

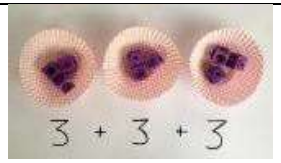
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	47 - 24 = 23 40 + 7 - 20 + 4 20 + 3 This will lead to a clear written column subtraction. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with regrouping once before moving onto subtractions where you need to regroup twice. Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to regroup one of my tens for ten ones. 	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing any regrouping.  When confident, children can find their own way to record the regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method.

	Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to regroup one hundred into ten tens.  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when regrouping and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. 

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition

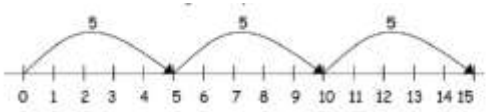


Use different objects to add equal groups.

3 x 4 or 3 lots of 4

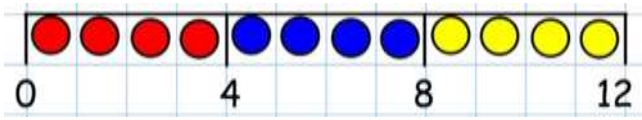
Use number lines to show repeated groups- 3 x 4

2 add 2 add 2 equals 6



5 + 5 + 5 = 15

Represent this pictorially alongside a number line e.g:



Use of a bar model for a more structured method.

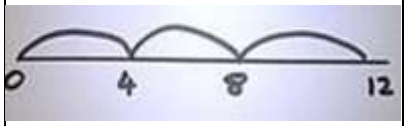
Write addition sentences to describe objects and pictures.

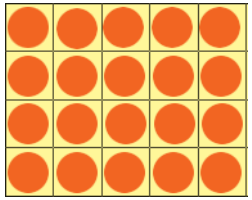
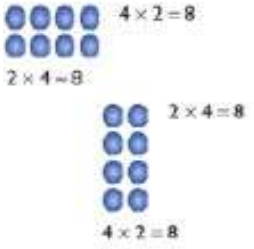
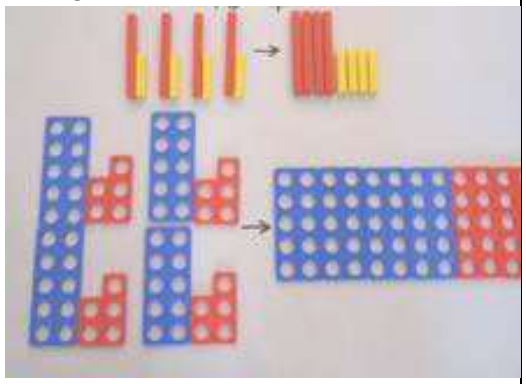
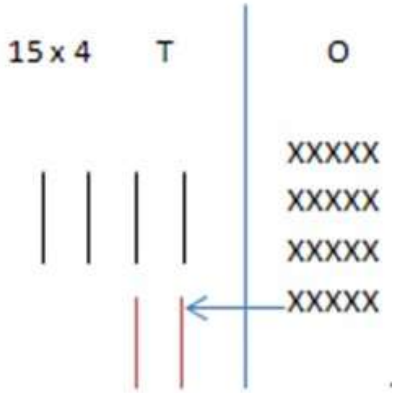
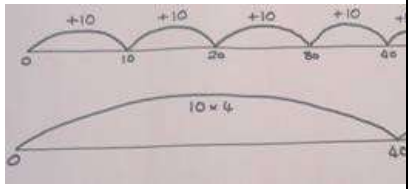
3 x 4

4 + 4 + 4

Abstract number line

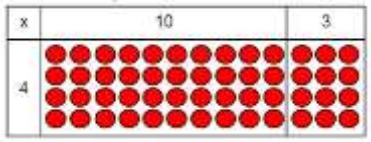
3 x 4 = 12



Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.    	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Partitioning to multiply	Use numicon, base 10, Cuisenaire rods 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:  	Children to be encouraged to show the steps they have taken 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 

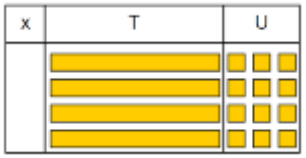
Grid Method

Show the link with arrays to first introduce the grid method.



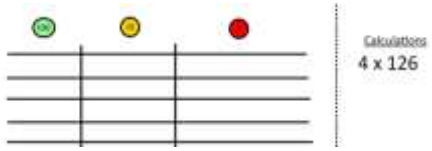
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

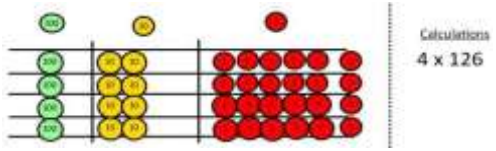


4 rows of 13

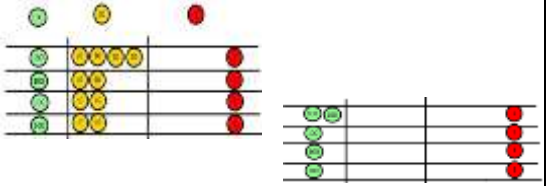
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



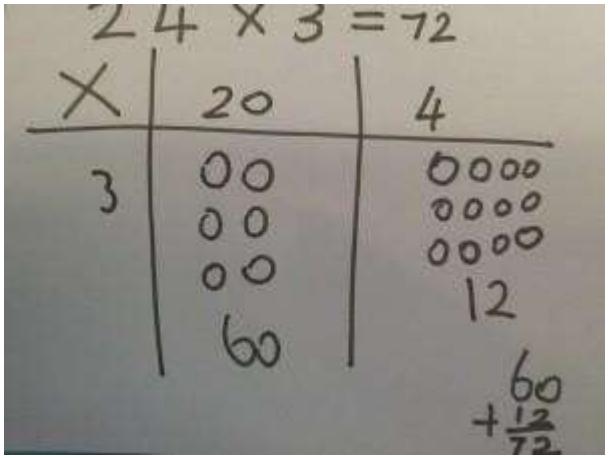
Add up each column, starting with the ones regrouping when needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

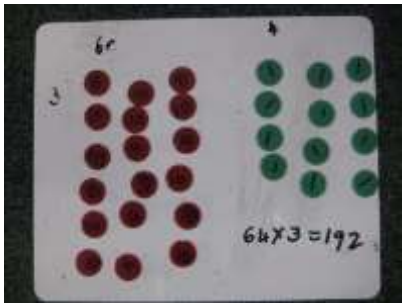
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

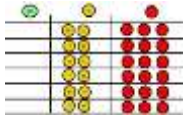
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.

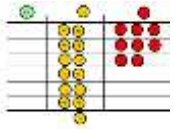


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

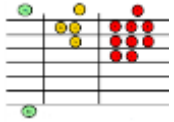
6 x 23



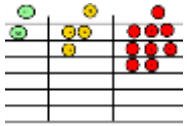
Step 1: get 6 lots of 23



Step 2: 6 x 3 is 18. Can I regroup? Yes! Ten ones for one ten….

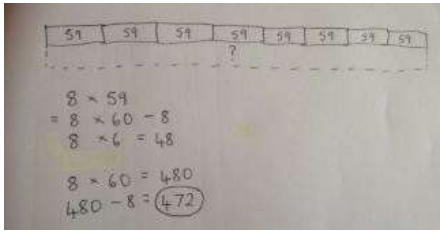


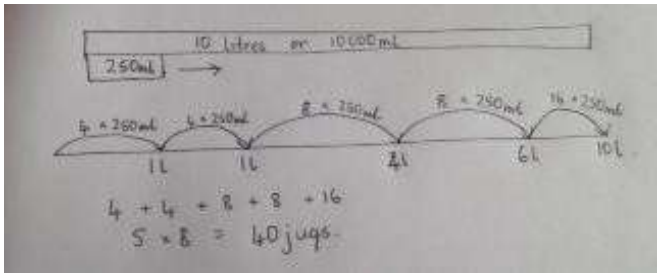
Step 3: 6 x 2 tens and my extra ten is 13 tens. Can regroup? Yes! Ten tens for one hundred…



Step 4- What do I have in each column?

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer.

```
  32
x 24
 ---
   8   (4 x 2)
 120   (4 x 30)
  40   (20 x 2)
 600   (20 x 30)
 ---
 768
```

This moves to the more compact method.

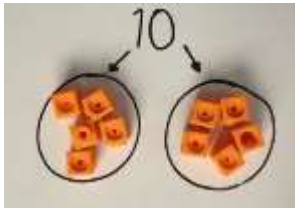
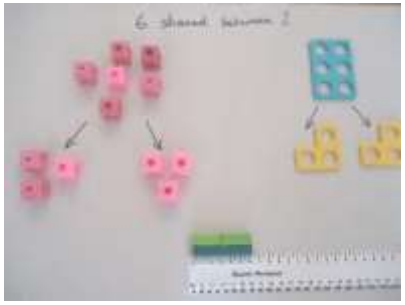
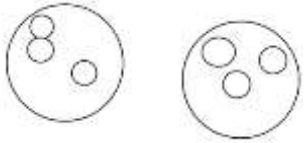
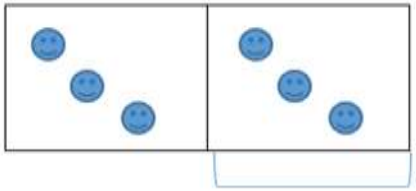
When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:

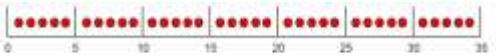
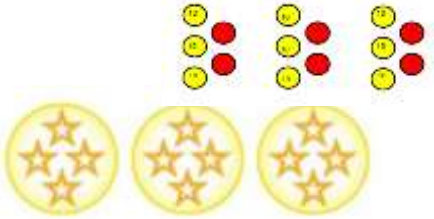
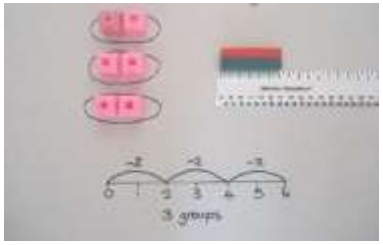
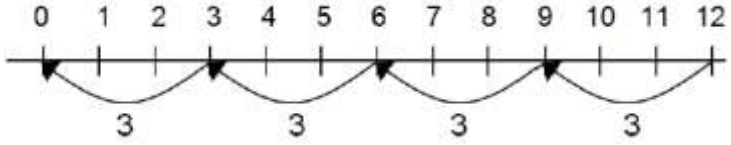
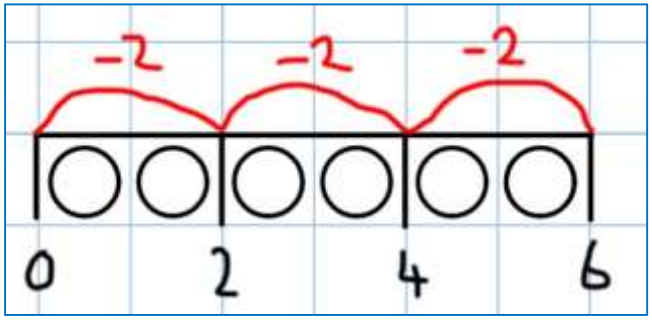
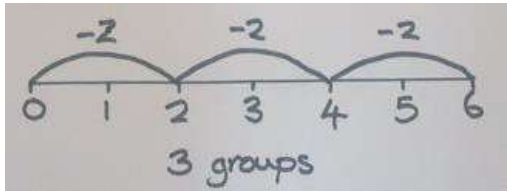
```
    1 2 4
  x   2 6
 -------
    7 4 4
  1   2
2 4 8 0
 -------
3 2 2 4
1 1
```

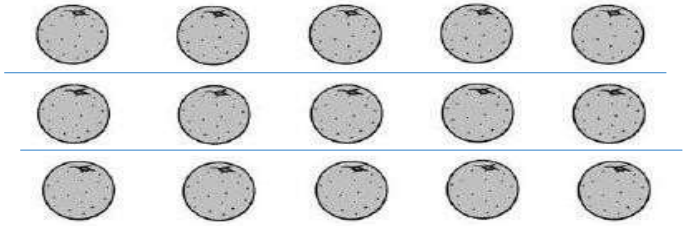
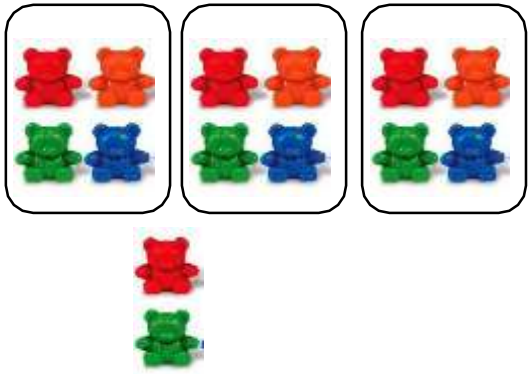
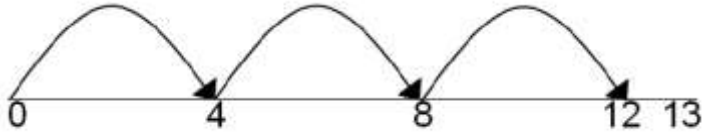
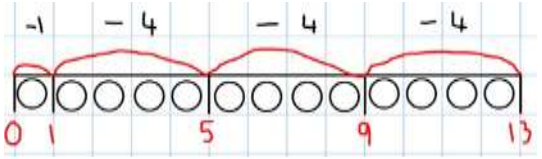
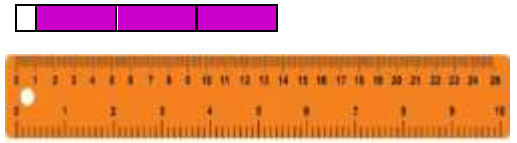
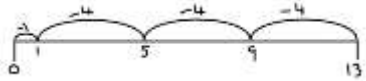
Answer: 3224

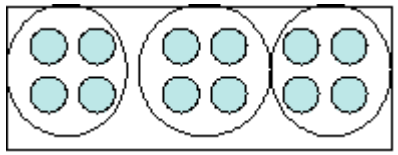
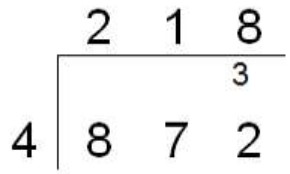
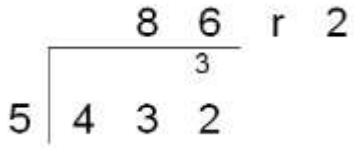
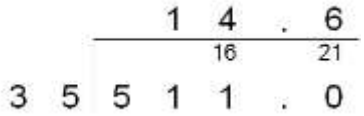
			To get 744 children have solved 6 x 124 To get 2480 they have solved 20 x 124 It is useful to show the grid method next to the standard written method so that the children can see how the two relate and that the mathematics involved is the same the only difference is the way it is written down.

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?  (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  6 ÷ 2 = 3  This can also be done in a bar so all 4 operations have a similar structure.	Share 9 buns between three people. 9 ÷ 3 = 3

Division as grouping and repeated subtraction	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32   6 ÷ 2	Use a number line to show jumps in groups. The number of jumps equals the number of groups. Make sure the children understand that division is repeated subtraction.    	28 ÷ 7 = 4 Divide 28 into lots of 7. How many lots are there?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.   	13 ÷ 4 – 3 remainder 1 Children to count their times tables facts in their heads  Complete written divisions and show the remainder using r. 

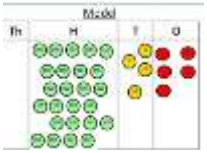
Short division	.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. 

Long division

2544 ÷ 12

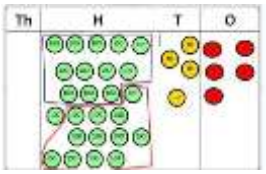
How many groups of 12 thousands do we have? None

Regroup 2 thousand for 20 hundreds.



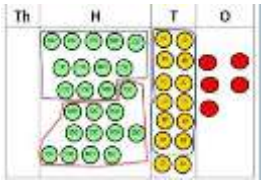
$$12\overline{)2544}\quad 0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



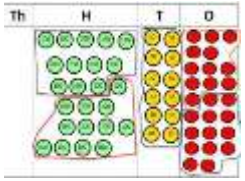
$$12\overline{)2544}\quad 02\ (24)\ 1$$

Regroup the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12\overline{)2544}\quad 021\ (24)\ 14\ 12\ 2$$

Regroup the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12\overline{)2544}\quad 0212\ (24)\ 14\ 12\ (24)\ 24\ 0$$

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

$$20\overline{)6365}\quad 0\ 3\ 1\ 8\ r5$$
$$60\downarrow\ 36\ 20\ 165\ 160\ 5$$

$$12\overline{)2544}\quad 0$$

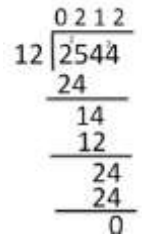
Step one- Regroup 2 thousand for 20 hundreds so we now have 25 hundreds.

$$12\overline{)2544}\quad 02\ (24)\ 1$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$12\overline{)2544}\quad 021\ (24)\ 14\ 12\ 2$$

Regroup the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I

			have, the 12 is how many I grouped and the 2 is how many tens I have left.  Regroup the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.